Two Stages of Economic Development

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December 12, 2015

Abstract

This paper suggests that the development process of a less-developed country can be divided into two stages with significantly different properties in terms of its structural endowments, production mode, income distribution and the driving force of economic growth, among others. The two stages of economic development have been indicated in the growth theory of macroeconomics and in the variety of "inflection point" theories in development economics, including Lewis's dual economy, Kuznets curve and middle income trap. We also construct a dynamic macroeconomic model to simulate the development process that reveals these two stages. Using this theory of two stages of economic development, we find that the current Chinese economy is at the intersection between the first and the second stages. This expresses the "new normal" of current Chinese economy.

Keywords: Development Process, Growth, Macrodynamic Model
JEL: O11 E10 C61 C62

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1 Introduction

In May 2015, president Xi used "new normal" to describe the current Chinese economy. The concept has now been popular in Chinese media and official documents. It also becomes a logic starting point in designing economic policy in China. Many economists in China also try to explain the "new normal" with many different perspectives.

In recent years, the concept of "middle income trap" proposed by World Bank (2007) has now been gradually circulated among economists since 2007. Although the middle income trap seems to be supported by empirical evidence, it has not been proved academically and supported by rigid economic analysis. The lack of theoretical supporting causes some economists to doubt about this concept.²

This paper tries to propose a theory called "two stages of economic development" (TSED). That theory is not only able to explain Xi’s "new normal" and World Bank’s "middle income trap", but also consistent with the growth theory in macroeconomics and the turning point theory in development economics, such as Lewis (1954) and Kuznets (1955).

In section 2, we shall use the growth theory in macroeconomics and turning point theory in development economics to explain the theory of "two stages of economic development" (TSED). This explanation does not rely on theoretical model. In section 3, we construct the model that allow us to derive the behavior function of investment, which is a key variable in the development process. This concept is used in the followed section where a theoretical model has been constructed to explain the development process of a less developed economy. Section 5 provides the analysis for the model, which allows us to see all the development features as aforementioned. Finally, in section 6, we try to argue that current Chinese economy has reached to the intersection of the first and the second stages of economic development, and thus provide a perspective with regard to Xi’s "new normal". The mathematical proof of the proposition in the text is provided in the appendix.

¹See evidence provided by World Bank (2007), Eichengreen et al. (2012, 2013), among others.
²See for instance, Han and Wei (2015), among others.
2 Two Stages of Economic Development, Indications from Growth Theory and Development Economics

The theory of two stages of economic development (TSED) was first proposed in Gong (2008, 2012) and modelled roughly in Gong (2013). Yet we shall remark that the theory indeed follows the logics indicated in the growth theory in macroeconomics and the variety of "turning point" theories in development economics, and thus it is constructed on solid economics foundation.

2.1 The Growth Theory in Macroeconomics

The main symbol of the fact that developing countries lag behind developed countries often rests with the difference in their per capita GDP (or output). According to the growth theory in macroeconomics, the lower GDP per capita in less developed economies results from two aspects. One is the lower capital per capita. This indicates that for the economy as a whole the production mode is mainly labor intensive. The other is the inadvanced technology, indicating that the total factor productivity is low in production function. Given these two reasons, the improvement of per capita output (GDP) can be realized by adopting the production modes either in capital intensive or in knowledge intensive.

In reality, the improvement in capital per capita often appears to be urbanization, that is, more and more labor leave from land and work with the capital (machines, facilities, etc.). The city can attract the labor from rural area just because of its continuous investment in building facilities. Thus, for the economy as a whole, the transformation of labor from rural to urban area is also a process of transformation of production mode from labor intensive to capital intensive. Thus, capital per capita is improved in the country.

However, according to the growth theory, there is a limit of increase in capital per capita and thus in GDP per capita at a given level of technology. In Figure 1, if the technology is kept at A, the economy starting with $k_0$, the initial capital per capita, will eventually move to its steady state $\bar{k}$.

\[ \theta = \frac{d + s}{1 + \delta}, \]

where $d$, $s$, and $l$ are respectively the depreciation rate, the saving ratio and the growth rate of labor supply.

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3See Solow (1956) or any intermediate textbook.
4Given the production function as $Y_t = (A_t L_t)^\alpha K_t^{1-\alpha}$ and the capital accumulation as $K_t = (1 - d)K_{t-1} + sY_t$, it is not difficult to prove that $\theta = \frac{d + s}{1 + \delta}$ and $\delta(A) = \frac{s^\alpha}{1 + s^\alpha}$.
intensive such as in $k_0$ to more capital intensive such as in $\bar{k}$. The GDP per capita is also improved to $\bar{y}$ as indicated in the figure. One can easily find that $(\bar{k}, \bar{y})$ is indeed the limit in capital per capita and output per capital, if technology is kept at $A$.

Once the limit has been achieved, only the improvement in technology can lead to further improvement in capital per capita and thus in GDP per capita. In Figure 1, if technology is improved to $A'$, capital per capita will increase to $\bar{k}'$ and thus GDP per capital will increase to $\bar{y}'$. We remark that the improvement in technology indicates that the production mode in the economy is towards knowledge intensive. It is, of course, the knowledge intensive that bring further increase in capital per capital and thus output per capital.

### 2.2 The Turning Points in Development Economics

There are a variety of turning point theories in development economics. The first is Lewis’ turning point, which is proposed by Lewis (1954) in his dual economy. In Lewis’s view, a less developed country has large or even infinite surplus labor in the early stages of economic development, and the economic society is separated into a relatively modern industrial (or urban) sector and
the traditional agricultural (or rural) sector, while a large number of surplus labor is stranded in the rural area. However, with the economic development, the surplus labor in the traditional agricultural sector is gradually absorbed by the modern industrial sector. The so-called Lewis’ turning point refers to the state in the development process that the labor surplus no longer exists. In this case, further growth will make the wage increase accelerated.

Kuznets curve put forward by Kuznets (1955) is another theory of turning point. According to this theory, the income distribution in the development process (measured by GDP per capita) of a country is improved after deterioration first. From this process, we can find the "turning point" (see point E in Figure 2) with regard to income distribution in the development process.

We believe that the "middle income trap" proposed by World Bank (2007) can also be regarded as another type of turning point theory. The so-called "middle income trap" means that when a country’s per capita GDP reaches to a range of middle income, the driving force of economic growth become inadequate, and thus the growth of per capita GDP could stagnate. In today’s world, a considerable part of developing countries achieved rapid economic growth in the earlier stage of economic development, but they are still caught in the "middle-income trap". "Middle income trap" also means that the "turning point" in the Kuznets curve is not easy to appear, or per capita GDP is not easy to grow further after reaching to the middle level E (see Figure 2).
2.3 The Two Stages of Economic Development

Economic development is a process with continuous improvement in GDP per capita. It is often accompanied by the continuous evolution of endowment structure, production mode, among others, which form the different stages of economic development. According to the previous discussion, we can divide the development process of a developing country into two stages (of course, the two stages might be partially overlapped, see Figure 3):

- The first stage: digesting surplus labor. In this stage, Lewis turning point has not appeared; the economy stays in the left side of Kuznets curve; it belongs to lower income country; the production mode is transforming from labor intensive to capital intensive.

- The second stage: catching up process of technology. In this stage, Lewis turning point has appeared; the economy is in the right side of Kuznets curve; the middle income trap has been escaped; the production mode is transforming from capital intensive to knowledge intensive.

Two remarks should be made here.

First, as discussed previously, digesting surplus labor is often the process of more rural labor moving into the city, that is, more and more labor left from land and corporated with the capital (machines, facilities, etc.). Therefore, for the country as a whole, digesting surplus labor is also the transformation process of production mode from labor intensive to capital intensive, which makes capital per capita continuously improving. Second, due to the existence of large surplus labor, the driving force of economic growth in the first stage includes not only the input of a large amount of surplus labor brought by capital investment, but also the inputs of technology. This means that the driving force is adequate for economic growth. In the second stage, the surplus labor is digested. Therefore, the source of growth in per capita GDP only comes from the inputs of technology. This indicates that the driving force of economic growth is relatively inadequate comparing to the first stage.

Table 1 compares the differences between these two stages.
2.4 Why Middle Income Trap?

The previous discussion only proposes that there are two stages in the development process experienced by a less developed country. But why is there a middle income trap between these two stages?

In the first place, we have already discussed, due to the exhaustance of surplus labor, the only driving force of economic growth in the second stage is the technological progress or the improvement in total factor productivity. Therefore, the driving force of economic growth has been reduced when the economy moves to the second stage.

Secondly, even the pattern of technological progress is also transformed and the improvement in total factor productivity is not easy in the second stage comparing to the first stage. In the first stage, due to the large difference in the development level, a less developed economy can simply import and imitate technologies from developed economies. Therefore the progress in technology need not relying on domestic research and development (R&D). Yet in the second stage, due to the less difference in development level, the import of technology is often restricted, and thus the progress in technology must rely on domestic R&D.\(^5\) This will make the technological progress in those middle income countries more difficult. We believe that this is the major reason of middle income trap despite the other possible reasons as discussed in literature.

\(^5\)See the discussion of technological progress in developing countries comparing with in developed countries in Acemoglu et al. (2006), Benhabib and Spiegel (1994), Vandenbussche et al. (2006) and Gong (2015) among others. We will return to this issue in more detail later in this paper.
The remaining of this paper is devoted to model the two stages of economic development as we have proposed.

3 Investment Behavior

In the development process, investment is a key economic variable since it accumulates the capital stock that attracts more labor to work with. As we have pointed out earlier, this is a main feature of development process, especially in the first stage. While creating capital stock (or capacity) for labor to work with, investment also creates demand. In this section, we first prove that under certain conditions, there exists a time-invariant optimum capacity utilization. Given this optimum capacity utilization, investment can then be understood as an adjustment to that optimum.

3.1 Technology

We begin our discussion with the production technology used in this paper. For a typical firm $j \in [0, 1]$ in period $t$, the production technology (or the input-output relation) is assumed to be in the form of the Cobb-Douglas described as follows:

$$Y_{j,t} = a (A_{j,t} L_{j,t})^\alpha K_{j,t-1}^{1-\alpha}$$

(1)

where, $Y_{j,t}$ is the output produced for $j$ at $t$; $K_{j,t-1}$ is the capital stock specific to $j$ measured at the end of period $t - 1$, so it provides the production facility in period $t$; $L_{j,t}$ is the labor employed by $j$; $A_{j,t}$ is a measure of labor efficiency (or technology), whose dynamics reflects technical progress; and $a$ is a time-invariant parameter.

This production function is very common in the literature. Here, we use $K_{j,t-1}$ rather than $K_{j,t}$ to enter the production function, merely to emphasize that the capital stock in period $t$ is fixed. Given the fixed capital stock, the capacity utilization can naturally be derived as follows.

3.2 Capacity Utilization

The input-output relation expressed in (1) implies that

$$A_{j,t} L_{j,t} = \left( \frac{Y_{j,t}}{aK_{j,t-1}^{1-\alpha}} \right)^{1/\alpha}$$

$$= Y_{j,t} \left( \frac{Y_{j,t}}{K_{j,t-1}} \right)^{\frac{1-\alpha}{\alpha}} \frac{1}{a^{1/\alpha}}$$
Define $B^\frac{1-\alpha}{\alpha} \equiv a^{1/\alpha}$. Thus, we find from the above that
\[ L_{j,t} = \frac{Y_{j,t}}{A_{j,t}} \left( \frac{Y_{j,t}}{BK_{j,t-1}} \right)^{\frac{1-\alpha}{\alpha}} \] (2)

Equation (2) can be understood as the demand function for labor, given the firm’s output $Y_{j,t}$, the capital stock $K_{j,t-1}$ and the labor efficiency $A_{j,t}$.

Next we shall define the capacity utilization $U_{j,t}$ as
\[ U_{j,t} \equiv \frac{Y_{j,t}}{BK_{j,t-1}} \] (3)

Further expressions might be needed here to capture the economic meaning of capacity utilization $U_{j,t}$ under the Cobb-Douglas production function that allows substitution between capital and labor. Suppose that in period $t$, the production facility represented by $K_{j,t-1}$ is given. The production activity can thus be understood as employing laborers to run the facility: the longer the facility runs, the larger the output produced. Therefore, we can define the capacity utilization $U_{j,t}$ as in (3), which roughly reflects the proportion of time that the facility runs in period $t$ (which generates output $Y_{j,t}$) over the normal working time available in a period (which generates the potential output, or capacity $BK_{j,t-1}$).  

Now, substituting (3) into (2), we obtain
\[ L_{j,t} = \frac{Y_{j,t}}{A_{j,t}} (U_{j,t})^{\frac{1-\alpha}{\alpha}} \] (4)

Equation (4) simply state that the demand for labor $L_{j,t}$ is used to run the facility to produce the output $Y_{j,t}$. Thus, $L_{j,t}$ is positively determined by output $Y_{j,t}$, and negatively by labor efficiency $A_{j,t}$, and is adjusted by capacity utilization $U_{j,t}$. There is no doubt that the longer the facility runs in a given period (or the higher the capacity utilization), the more labor is needed to produce additional output.

### 3.3 Cost Function

Given the definition of capacity utilization $U_{j,t}$ as expressed in (3), we find that the production cost can also be understood as a function of $U_{j,t}$. Let

\[ BK_{j,t} = \frac{Y_{j,t}}{U_{j,t}} \] (5)
$C_{j,t}$ denote the total cost in real terms for firm $j$ at $t$, and let $W_{j,t}$ denote the real wage rate paid by the firm. Ignoring the other intermediate input (such as raw materials, etc.), the total cost of the firm can be written as

$$C_{j,t} = L_{j,t}W_{j,t} + vK_{j,t-1}$$

where the labor cost $L_{j,t}W_{j,t}$ can be regarded as a variable cost (because, as shown below, it will vary with the produced output $Y_{j,t}$), and $vK_{j,t-1}$ is a fixed cost, which will not vary with output $Y_{j,t}$ but with the capital stock $K_{j,t-1}$. Now, expressing $L_{j,t}$ in terms of (4), we obtain

$$C_{j,t} = W_{j,t}Y_{j,t}A_{j,t}(U_{j,t})^{\frac{1-\alpha}{\alpha}} + vK_{j,t-1}$$

Suppose $\frac{W_{j,t}}{A_{j,t}} = \omega$, that is, the real wage $W_{j,t}$ increases at the same rate as the labor efficiency $A_{j,t}$. Thus, given the total cost as in (5), the marginal cost $C'_{j,t} = \frac{\partial C_{j,t}}{\partial Y_{j,t}}$ and the average cost $c_{j,t} = \frac{C_{j,t}}{Y_{j,t}}$ can be written as

$$C'_{j,t} = \frac{\omega}{\alpha} \left( U_{j,t} \right)^{\frac{1-\alpha}{\alpha}}$$

$$c_{j,t} = \omega \left( U_{j,t} \right)^{\frac{1-\alpha}{\alpha}} + \frac{v}{B} \left( U_{j,t} \right)^{-1}$$

Above, $\omega \left( U_{j,t} \right)^{\frac{1-\alpha}{\alpha}}$ can be regarded as the average variable cost (AVC) and $\frac{v}{B} \left( U_{j,t} \right)^{-1}$ as the average fixed cost (AFC).

### 3.4 Optimum Capacity Utilization when Capital Stock is not Adjustable

It is useful to derive the level of capacity utilization that minimizes the average cost. From (7), the first-order condition for this minimization problem can be written as

$$\frac{1-\alpha}{\alpha} \omega \left( U_{j,t} \right)^{\frac{1-\alpha-1}{\alpha}} - \frac{v}{B} \left( U_{j,t} \right)^{-2} = 0$$

Solving the above equation for $U_{j,t}$, we obtain

$$U_{j,t}^* = \left[ \frac{\alpha v}{\omega (1-\alpha) B} \right]^\alpha$$

This can be regarded as the optimum capacity utilization when the capital stock is given (or not adjustable). We find that it is indeed time-invariant.
Let $C_{j,t} = c_{j,t}$. From (6) and (7), we find that
\[
\frac{\omega}{\alpha} (U_{j,t})^{1-\alpha} = \omega (U_{j,t})^{1-\alpha} + \frac{\varphi}{B} (U_{j,t})^{-1}
\]
Solving the above equation for $U_{j,t}$, we again obtain (8). Therefore, the level of capacity utilization that minimizes the average cost, as expressed in (8), is also the level at which the marginal cost cuts the average cost. Figure 4 provides the different costs as a function of capacity utilization.

The above discussion seems to suggest that the standard firm theory in microeconomics with regard to the cost function still holds in terms of capacity utilization. In particular, given the capital stock $K_{j,t-1}$, the marginal cost and the variety of average costs can all be expressed as a function of capacity utilization $U_{j,t}$. In addition, if $\frac{W_{j,t}}{A_{j,t}} = \omega$, the functions are also time-invariant. This also indicates that the optimum level of capacity utilization that minimizes the average cost can be a constant (see equation (8)).

### 3.5 Optimum Capacity Utilization when Capital Stock is Adjustable

The capacity utilization expressed in (8) can be understood as the optimum capacity utilization when capital stock is given (not adjustable). Suppose now that the average cost is too high due to the high capacity utilization. In this case, investment is needed to expand the capacity to reduce the average cost.
Investment is constructed for future capacity; therefore, we assume that the firm has been given a sequence of expected demands \( E \{ Y_{j,t+k} \}_{k=0}^{\infty} \), a sequence of technologies \( E \{ A_{j,t+k} \}_{k=0}^{\infty} \), and a sequence of real wages \( E \{ W_{j,t+k} \}_{k=0}^{\infty} \), among others, when making an investment decision in period \( t \). The investment decision problem can thus be expressed as the choice of a sequence of investments \( \{ I_{j,t+k} \}_{j=0}^{\infty} \) such that

\[
\max_{\{ I_{j,t+k} \}_{k=0}^{\infty}} \sum_{k=0}^{\infty} \beta^k \left[ P_{j,t+k}Y_{j,t+k} - P_{t+k}c_{j,t+k}Y_{j,t+k} - (1+r)P_{t+k}I_{j,t+k} \right] \quad (9)
\]

subject to

\[
K_{j,t+k} = (1 - d_j)K_{j,t+k-1} + I_{j,t+k} \quad (10)
\]

where \( E \) is the expectation operator; \( \beta \) is the discount factor; \( r \) can be regarded as the interest rate that reflects the firm’s opportunity cost of investment; \( P_{j,t+k} \) is the price of the product produced by firm \( j \); \( P_{t+k} \) is the aggregate price level; \( c_{j,t+k} \) is the average cost, expressed by (7); and \( d_j \) is the depreciation rate. Equation (10) can be regarded as the process of capital accumulation.

Proposition 1 provides the solution to this optimization problem.

**Proposition 1** Suppose \( E \left[ \frac{W_{j,t+k}}{A_{j,t+k}} \right] = \omega \) and \( E \left[ \frac{P_{t+k}}{P_{t+k-1}} \right] = \pi \). Then, the problem in (9) - (10) with \( c_{j,t+k} \) given by (7) allows us to obtain

\[
U_{j,t+k}^* = U_j^* = \left( \frac{\alpha [(1+r)/(\beta \pi) - (1+r)(1-d_j) + v_j]}{(1-\alpha)B\omega} \right)^\alpha \quad (11)
\]

where \( k = 1, 2, 3, \ldots \).

The proof of this proposition is given in the appendix.

Equation (11) is quite similar to equation (8) when the capital stock is given (or not adjustable). Consider \( \beta = 1 \), \( r = 0 \), \( \pi = 1 \), and \( d_j = 0 \), so that we return to a one-period decision. In this case, the two equations (11) and (8) coincide.

### 3.6 Investment without Financial Constraint

Given an optimum level of capacity utilization \( U_j^* \), as expressed in (11), we shall now consider how the investment should be made. Because the investment carried in period \( t \) creates the capital stock \( K_{j,t} \) that serves the
capacity for period $t + 1$, the optimum investment, denoted as $I^*_{j,t}$, should satisfy

$$
\frac{E[Y_{j,t+1}]}{B[(1 - d_j)K_{j,t-1} + I^*_{j,t}]} = U^*_j
$$

where the left side of the equation can be understood as the expected capacity utilization for period $t + 1$. Resolving this equation for $I^*_{j,t}$, we obtain

$$
I^*_{j,t} = \frac{E[Y_{j,t+1}]}{BU^*_j} - (1 - d_j)K_{j,t-1}
$$

Dividing both sides by $K_{j,t-1}$, we obtain

$$
\frac{I^*_{j,t}}{K_{j,t-1}} = \frac{E[Y_{j,t+1}]}{BU^*_j K_{j,t-1}} - (1 - d_j)
= -(1 - d_j) + \left(\frac{E[y_{j,t+1}]}{U^*_j}\right)U_{j,t}
$$

where $E[y_{j,t+1}]$ is the expected gross growth rate of product $j$.

We assume that at the very beginning of period $t$ when investment decision is made, the firm may not observe its market demand in $t$ and thus the capacity utilization $U_{j,t}$. In this case, $U_{j,t}$ in (13) can simply be regarded as the expected capacity utilization of $U_{j,t}$ given the information in $U_{j,t-1}$. Suppose that $E[U_{j,t}] = U_{j,t-1}$ and $E[y_{j,t+1}] = y_j$. We find that (13) can be re-written as

$$
\frac{I^*_{j,t}}{K_{j,t-1}} = -(1 - d_j) + \frac{y_j}{U^*_j}U_{j,t-1}
$$

This equation indicates that the investment rate $\frac{I^*_{j,t}}{K_{j,t-1}}$ depends on the observed capacity utilization: the higher the capacity utilization, the higher the investment rate.

The discussion here with regard to investment allows us to obtain the following interpretation of investment decision:

Suppose the investment is divisible. Whatever the level of expected demand, $E[Y_{j,t+1}]$, the purpose of investment $I_{j,t}$ is simply to adjust the capital stock $K_{j,t}$ to the level at which $E[Y_{j,t+1}]/(BK_{j,t})$ is equal to $U^*_j$, as in (11), which minimizes the average cost of production.
3.7 Investment with Financial Constraint

Next, we shall consider the effects of monetary policy on the economy. It is apparent that it should affect investments. The investment as we considered in (14) is the optimum investment desired by firm \( j \) if there is no financial restriction. A financial restriction may affect the investment through two channels: the interest rate and the credit supply. Consistent with the money supply rule adopted in this paper (see equation (28) later in this paper), we consider the credit supply.

Suppose that our representative firm \( j \) is able to acquire a loan from a commercial bank (in real terms) up to \( M_{j,t}^{1} \) for its investment.\(^7\) This indicates that the firm’s investment under credit constraint can be written as

\[
I_{j,t} = \begin{cases} 
I_{j,t}^{*} & I_{j,t}^{*} < \Delta M_{j,t-1} \\
\Delta M_{j,t-1} & \text{otherwise}
\end{cases}
\]  

(15)

Let \( \Delta M_{t-1} \) denote the total additional money (or credit) from the commercial bank system in period \( t-1 \). This money supply is targeted by the monetary authority, and it is the amount of money that a commercial bank can lend to finance the investment. Given \( \Delta M_{t-1} \), we write \( \Delta M_{j,t-1} \) as

\[
\Delta M_{j,t-1} = l_{j} \Delta M_{t-1} \tag{16}
\]

where \( l_{j} \in [0, 1) \) is the proportion of total credit allocated to \( j \).

Under this credit plan, the firm makes its investment decisions according to (15). Summing all \( I_{j,t} \)'s, we get the aggregate investment \( I_{t} \):

\[
I_{t} = \int_{0}^{1} I_{j,t} dj
\]

(17)

Depending on the credit ratio \( l_{j} \) assigned to the firm, we find that for some \( j \)'s, investments are bounded, that is, \( I_{j,t} = \Delta M_{j,t-1} \); whereas for others, investments are at the optimum, that is, \( I_{j,t} = I_{j,t}^{*} \), where \( I_{j,t}^{*} \) is given by (14). Re-arranging the index of the firms such that the first \( n_{1} \) proportion of the firms are bounded, we can write (17) as

\[
I_{t} = \phi \Delta M_{t-1} + \int_{n_{1}}^{1} \left[ -(1 - d_{j})K_{j,t-1} + \left( \frac{1 + y_{j}}{U_{j}^{*}} \right) U_{j,t-1}K_{j,t-1} \right] dj
\]

(18)

where \( \phi = \int_{0}^{n_{1}} l_{j} dj \). Under the identical assumption of a representative agent, the above equation can be re-written as

\[
I_{t} = \phi \Delta M_{t-1} - (1 - d) \int_{n_{1}}^{1} K_{t-1} dj + \frac{1 + y}{U_{t}^{*}} U_{t-1} \int_{n_{1}}^{1} K_{t-1} dj
\]

\(^7\)Here, we assume that it is the money supply (or credit) in period \( t-1 \) that is used for financing the investment in period \( t \).
Dividing both sides of the above equation by $K_{t-1}$, the aggregate capital stock, we obtain from the above

$$
\frac{I_t}{K_{t-1}} = \phi \Delta M_{t-1} - (1 - d)n_k + \frac{(1 + y)n_k}{U^*} U_{t-1}
$$

(19)

where $n_k$ can be regarded as the proportions of capital stock from those unrestricted firms:

$$
n_k \equiv \frac{\int_{n_1}^1 K_{j,t-1} dj}{K_{t-1}}
$$

Here, we assume this proportion to be time-invariant.

To ensure our analysis is tractable, we assume a linear relationship between the aggregate money supply and the aggregate capital stock $K_{t-1} = \eta M_{t-2}$. As we are working with aggregate variables, the rationality of this linear relationship is considered more from a statistical view point.\(^8\)

Given this linear proposition, we can now re-write our aggregate investment function (19) as

$$
\frac{I_t}{K_{t-1}} = -\xi_i + \xi_u U_{t-1} + \xi_m (m_{t-1} - p_{t-1})
$$

(20)

where $m_{t-1} - p_{t-1} \approx M_{t-1}/M_{t-2}$ is the approximate gross growth rate of the credit supply in real terms. Note that here, $m_t$ is the nominal growth rate of the credit supply, which is targeted by the monetary authority. The parameters $\xi_i$, $\xi_u$ and $\xi_m$ are given by

$$
\xi_i = (1 - d)n_k, \quad \xi_u = \frac{(1 + y)n_k}{U^*}, \quad \xi_m = \phi/\eta.
$$

4 The Model

Given the investment function as described in the last section, we shall now build our model that reflects the development process from a less-developed dual economy.

4.1 The Structure Form of the Model

The model includes the following equations:

\(^8\)Gong and Lin (2008) and Gong (2013) estimate the investment function using data from China and show that the estimation is statistically significant.
\[ Y_t = I_t + C_t \]  
(21)

\[ C_t = (1 - s)Y_t \]  
(22)

\[ U_t = \frac{Y_t}{BK_{t-1}} \]  
(23)

\[ K_t = (1 - d)K_{t-1} + I_t \]  
(24)

\[ L_t = \frac{Y_t}{L_t} \]  
(25)

\[ L_t^s = (1 + l)L_{t-1}^s \]  
(26)

\[ N_l = \frac{L_t}{L_t^s} \]  
(27)

\[ m_t = \kappa(p^* - p_{t-1}) - m_{t-1} \]  
(28)

\[ \frac{I_t}{K_{t-1}} = -\xi + \xi_u U_{t-1} + \xi_m (m_{t-1} - p_{t-1}) \]  
(29)

\[ p_t = \beta_p + \beta_w w_t + \beta_u U_{t-1} \]  
\[ \beta_w, \beta_u, \beta_x > 0 \]  
(30)

\[ w_t = \alpha_w + \alpha_pp_t + \alpha_{n,t} N_{t-1} + \alpha_x x_t \]  
\[ \alpha_p, \alpha_n, \alpha_x > 0 \]  
(31)

\[ \alpha_{n,t} = \begin{cases} 
0, & 0 \leq N_{t-1} < N^b \\
-a + bN_{t-1}, & a/b \leq N_{t-1} < N^* \\
c, & N^* \leq N_{t-1} < +\infty 
\end{cases} \]  
(32)

\[ A_t - A_{t-1} = \begin{cases} 
\theta_f[A_{t-1}^f(1 - \epsilon) - A_{t-1}] + \theta_a A_{t-1}, & \text{if } A_{t-1}^f(1 - \epsilon) > A_t^- \\
\theta_a A_{t-1}, & \text{otherwise} 
\end{cases} \]  
(33)

\[ A_t^f = (1 + x^f)A_{t-1}^f \]  
(34)

Above, \( Y_t \) is referred to the output, which is composed of consumption \( C_t \) and investment \( I_t \); \( U_t \) is defined to be the capacity utilization; \( K_t \) is the capital stock; \( L_t \) is the employment; \( A_t \) is technology; \( L_t^s \) is the labor supply with the growth rate \( l \); \( N_l \) is the employment rate; \( \alpha_{n,t} \) is the growth rate of money supply; \( \alpha_{n,t} \) is the inflation rate; \( I_t/K_{t-1} \) can be regarded as the investment rate; \( w_t \) is the growth rate of nominal wage; \( x_t \) is the growth rate of technology; \( \alpha_{n,t} \) is a time-varied parameter and finally \( A_{t-1}^f \) is the technology in a frontier country that we used for comparison. All variables are assumed to be measured at the aggregate level.

Apparently, equation (21) is a definition of aggregate output in a closed and simple economy; (22) is a consumption function as usual; (23) is a definition of capacity utilization, whose economic meaning has been discussed in detail in the last section; (24) reflects the accumulation of capital stock as usual; (25) is the demand for labor, which we have also discussed in the last section (see equation (4)); (26) reflects the dynamics of labor supply,
which follows a constant growth rate \( l \); (27) is the definition of employment rate; (28) reflects the behavior of monetary policy whose only target is at the inflation rate \( p^* \); (29) is the behavior function of investment rate that has been derived in the last section; (30) and (31) are the dual Philips curves as discussed widely in the literature such as Flaschel, et al. (2001, 2002) and Fair (2000), among others. The key difference here is that we have assumed a time-varied parameter \( \alpha_{n,t} \) whose behavior given in (32) that reflects the development process from a less-developed dual economy, and finally, (33) and (34) reflects the dynamics of technology in our domestic economy and the frontier economy.

Remarks should be made first with respect to dynamics of technology. It is well known that for a frontier developed country, its technological progress must rely on its own research and development (R&D). The standard endogenous growth theory (or new growth theory) such as Romer (1990) and Lucas (1988), among others, are thus often applied to frontier developed economies. The technological progress via R&D is often difficult in the sense it will request larger investment with higher risky, and large high quality of human resource. More importantly, it also requests the institutional environment that provides the incentive and protection for R&D. For less-developed economies, such requirements are often difficult to meet. Yet a less-developed economy can take the advantage of huge distance from frontier economies in technological level to import technology. Such type of technological progress is more economic and less risky though what imported cannot be those most advanced technology, but are those no longer suitable in the frontier economy. Therefore, in equation (33), we classify the technological progress, \( A_t - A_{t-1} \) of our less-developed economy into two types: one is from importing technology, \( \theta_f[A_{t-1}^f(1-\epsilon) - A_{t-1}] \), the other is from domestic R&D, \( \theta_a A_{t-1} \). In particular, \( \epsilon \) can be regarded as the blocking rate so that as long as the distance \( A_{t-1}^f - A_{t-1} \) is less than certain proportion \( \epsilon A_{t-1}^f \), there will be no import of technology; \( \theta_f \) can be regarded import coefficient while \( \theta_a \) is the growth rate of technology from domestic independent R&D. There is no doubt that both \( \theta_f \) and \( \theta_a \) depends on the institution, human capital level, among others, in the domestic economy.\footnote{See Benhabib and Spiegel (1994), Acemoglu et al. (2006), Vandenbussche et al. (2006) and Gong (2015) among others.}

Second, in our dual Philips curves (30) and (31), we have assumed that the dynamics of price and wage are based on the fairly symmetric assumptions on the causes of price and wage inflation. Both of them are driven, on one hand, by the demand pressure components \( U_{t-1} \) or \( N_{t-1} \), and on the other hand, by the cost push terms measured by \( p_t, w_t \) and \( x_t \) in the right side
of (30) and (31). Such a way of price and wage dynamics is also consistent with the recent sticky pricing of New Keynesian economics in the sense that they do response to the market pressure but not necessarily clear the markets at every period.\textsuperscript{10} Apparently, for a developing economy, the state of non-clearing market is more evident comparing to developed economy.

The economic meaning of equation (32) can be expressed as follows. When the economy is in the dual economy, a large amount of surplus labors exists in rural area. In our one sector model, this can be regarded as low employment rate (lower than $N^b$). In this case, it is reasonable to assume that the dynamics of wage rate $w_t$ does not response to the labor market status reflected by $N_t$, that is, $\alpha_{n,t} = 0$. With the economic development, more surplus labors are absorbed and employed in the city. This indicates that $N_t$ is increasing. When $N_t$ is increasing to the threshold $N^b$, the wage rate become sensitive to the labor market status $N_t$, and thus $\alpha_{n,t} > 0$. However, we shall assume that there is an upper bound of $\alpha_{n,t}$, which is assumed to be $c$, when the economy is close to the full-employment. In Figure 5, we portraint the function of $\alpha_{n,t}$.\textsuperscript{11}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{The Curve of $\alpha_{n,t}$}
\end{figure}

\footnotetext[10]{We borrow the name of "sticky" also because the price equation we define here is similar to NKPC (New Keynesian Philips Curve). It is well known (e.g. from Gali, 2007) that NKPC can be defined as $\pi_t = \beta E[\pi_{t+1}] + \kappa \bar{y}_t$, where $\pi_t$ is the gross inflation rate, $\bar{y}_t$ the output gap, similar to our capacity utilization. Thus if we assume that $E[\pi_{t+1}]$ is a linear projection from $w_t$, with $w_t$ to be the wage inflation, this will be the same as the price equation in our dual Philips curves.}

\footnotetext[11]{We should remark that similar non-linearity has also been posed by Fair (2000).}
4.2 The Intensive Form of the Model

The following proposition regards the intensive form of the model.

**Proposition 2** Let $i_t \equiv \frac{I_t}{K_{t+1}}$, $h_t \equiv \frac{A_t}{A_{t-1}}$, $K_t \equiv \frac{K_t - K_{t-1}}{K_{t-1}}$, and $y_t \equiv \frac{Y_t - Y_{t-1}}{Y_{t-1}}$. Then, the structural form of the model (21) - (34) can be reduced to the following intensive form.

\[
\begin{align*}
    m_t &= \kappa(p^* - p_{t-1}) - m_{t-1} \\
    i_t &= \xi + \frac{\xi}{sB}i_{t-1} + \xi(m_{t-1} - p_{t-1}) \\
    p_t &= \beta_0 + \beta_1 i_{t-1} + \beta_2 \alpha_{n,t} N_{t-1} + \beta_3 x_t \\
    N_t &= \frac{1 - d + i_{t-1}}{1 + x_t(1 + t)} \left( \frac{i_t}{i_{t-1}} \right)^{\frac{1}{n}} N_{t-1} \\
    w_t &= \alpha_0 + \alpha_1 h_{t-1} + \alpha_2 \alpha_{n,t} N_{t-1} + \alpha_3 x_t \\
    k_t &= -d + i_t \\
    y_t &= \frac{i_t(1 - d + i_{t-1})}{i_{t-1}} - 1 \\
    x_t &= \begin{cases} \\
        \theta_f(1 - \epsilon)h_{t-1} + (\theta_a - \theta_f), & \text{if } h_{t-1} > \frac{1}{1 - \epsilon} \\
        \theta_a, & \text{otherwise} \\
    \end{cases} \\
    h_t &= \begin{cases} \\
        \left( \frac{1 + x_f}{1 - \epsilon} \right)h_{t-1}, & \text{if } h_{t-1} > \frac{1}{1 - \epsilon} \\
        \left( \frac{1 + x_f}{1 + \theta_a} \right) h_{t-1}, & \text{otherwise} \\
    \end{cases}
\end{align*}
\]

where $\alpha_{n,t}$ in (39) is governed by (32) and

\[
\begin{align*}
    \beta_0 &= \frac{\beta_p + \beta_w \alpha_w}{1 - \beta_w \alpha_p}, & \beta_1 &= \frac{\beta_w}{sB(1 - \beta_w \alpha_p)}, \\
    \beta_2 &= \frac{\beta_w}{1 - \beta_w \alpha_p}, & \beta_3 &= \frac{\beta_w \alpha_x}{1 - \beta_w \alpha_p} \\
    \alpha_0 &= \frac{\alpha_w + \alpha_p \beta_p}{1 - \alpha_p \beta_w}, & \alpha_1 &= \frac{\alpha_p \beta_u}{(1 - \alpha_p \beta_w) sB}, \\
    \alpha_2 &= \frac{1}{1 - \alpha_p \beta_w}, & \alpha_3 &= \frac{\alpha_x}{1 - \alpha_p \beta_w}
\end{align*}
\]

The proof of this proposition is given in the appendix of this paper.

Apparently, the intensive form of the model is highly dimensional. Yet, the variable $h_t$ is autonomously determined via (43). Given the determination of $h_t$, $x_t$ is determined by via (42), and thus enter (37), (38) and (39) as an exogenous variable. The system is also recursive in the sense that
the variables such as $w_t$, $k_t$ and $y_t$ can be derived given the dynamics in $(m_t, i_t, p_t, N_t)$, indicating the system is 4-dimensional in $(m_t, i_t, p_t, N_t)$.

One can easily find that our model involves three sub-systems, which may include different dynamics and steady states depending on labor market status $N_t$. If $N_t$ is in the range of $[0, N^b)$, the system is 3-dimensional in the space of $(m_t, i_t, p_t)$ with the equations including (35), (36) and

$$p_t = \beta_0 + \beta_1 i_{t-1} + \beta_3 x_t$$  \hfill (44)$$

Given the dynamics of $(m_t, i_t, p_t)$, $N_t$, $k_t$ and $y_t$ are determined by (38), (40) and (41) respectively while

$$w_t = \alpha_0 + \alpha_1 i_{t-1} + \alpha_3 x_t$$  \hfill (45)$$

We call this sub-system as system 1.

When $N_t$ is in the range of $[N^b, N^*)$, the system become 4-dimensional in the space of $(m_t, i_t, p_t, N_t)$ with the equations including (35), (36), (38) and

$$p_t = \beta_0 + \beta_1 i_{t-1} + \beta_2 a N_{t-1} + \beta_3 b N_{t-1}^2 + \beta_3 x_t$$  \hfill (46)$$

Given the dynamics of $(m_t, i_t, p_t, N_t)$, $k_t$ and $y_t$ can again be derived via (40) and (41) respectively while $w_t$ is determined via

$$w_t = \alpha_0 + \alpha_1 i_{t-1} + \alpha_2 a N_{t-1} + \alpha_2 b N_{t-1}^2 + \alpha_3 x_t$$  \hfill (47)$$

We may call this sub-system as system 2.

The system 3 is referred to the trajectory of $N_t$ moving in the range of $[N^*, +\infty)$. It is similar to the system 2 except that (46) and (47) are replaced by

$$p_t = \beta_0 + \beta_1 i_{t-1} + \beta_2 c N_{t-1} + \beta_3 x_t$$
$$w_t = \alpha_0 + \alpha_1 i_{t-1} + \alpha_2 c N_{t-1} + \alpha_3 x_t$$

4.3 The steady states

The derivation of the steady states of $x_t$ and $h_t$ will be illustrated in the next section. Here we simply assume that $x_t$ has a steady state denoted as $\bar{x}$. Given $\bar{x}$, the steady states of the model (35) - (41) are given in the following proposition.

**Proposition 3** Let $(\bar{m}, \bar{p}, \bar{w}, \bar{i}, \bar{N}, \bar{k}, \bar{y})$ denote respectively the steady states of $(m_t, p_t, w_t, i_t, N_t, k_t, y_t)$. Given that the steady state of $x_t$ is equal to $\bar{x}$, we find that the system composed of (35) - (41) has the following economically meaningful steady states:
• For system 1

\begin{align*}
\bar{p} &= p^* \\
\bar{i} &= \frac{p^* - \beta_0 - \beta_3\bar{x}}{\beta_1} \\
\bar{m} &= \frac{1}{\xi_m} \left[ \left( 1 - \frac{\xi_0}{sB} \right) \bar{i} - \xi + \xi_mp^* \right] \\
\bar{w} &= \alpha_0 + \alpha_1\bar{i} + \alpha_3\bar{x} \\
\bar{k} &= \bar{y} = -d + \bar{i}
\end{align*}

while \( N_t \) moves at the steady state according to

\[ N_t = \frac{1 - d + \bar{i}}{(1 + \bar{x})(1 + \bar{l})} N_{t-1} \]  

• For system 2, the steady state can be written as

\begin{align*}
\bar{i} &= \bar{x} + l + d + \bar{x}l \\
\bar{N} &= \frac{1}{2\beta_2b} \left( a\beta_2 + \sqrt{(-a\beta_2)^2 - 4\beta_2b(\beta_0 + \beta_3\bar{x} + \beta_1\bar{i} - p^*)} \right) \\
\bar{w} &= \alpha_0 + \alpha_1\bar{i} + \alpha_2a\bar{N} + \alpha_2b\bar{N}^2 + \alpha_3\bar{x}
\end{align*}

while \( \bar{p}, \bar{m}, \bar{k} \) and \( \bar{y} \) are the same as in system 1.

• For system 3, the steady state can be written as

\begin{align*}
\bar{N} &= -\frac{\beta_0 + \beta_3\bar{x}}{\beta_2c} - \frac{\beta_1\bar{i}}{\beta_2c} + \bar{p} \\
\bar{w} &= \alpha_0 + \alpha_1\bar{i} + \alpha_2c\bar{N} + \alpha_3\bar{x}
\end{align*}

while \( \bar{i}, \bar{p}, \bar{m}, \bar{k} \) and \( \bar{y} \) are the same as in system 2.

The proof of this proposition is simple and thus not provided in this paper.

Several remarks should be made here with regard to the proposition.

First, for system 1, we find that \( N_t \) has no steady state: it will move according to (53) when other variables such as \( i_t \) are at their steady states. This indicates that as long as

\[ \frac{1 - d + \bar{i}}{(1 + \bar{x})(1 + \bar{l})} > 1 \]  

(57)
$N_t$ will increase and thus the system will eventually switch to system 2. Substituting (49) into (57), we find that condition (57) will be satisfied if

$$\frac{p^* - \beta_0 - \beta_3 \bar{x}}{\beta_1} - d > \bar{x} + l + \bar{x}l$$

This indicates that the government can at least do something (such as setting a higher target of inflation rate) to switch the economy into system 2 with higher employment rate, that is, $N_t > N^b$.

Second, from (52), we have found that the gross growth rate of $K_t$ and $Y_t$ are equal to $1 - d + i$. Using (54) to express $i$, we find that gross growth rate in system 2 and 3 is equal to $(1 + \bar{x})(1 + l)$. In other words,

$$1 + \bar{y} = 1 + \bar{k} = (1 + \bar{x})(1 + l) \quad (58)$$

This indicates that the economy will eventually grow at the natural rate, which approximately is equal to the sum of growth rates in technology and labor supply.\(^{12}\)

Third, from condition (52) and (57), it is also not difficult to find that for the system 1 to switch to system 2 and 3, the growth rate of $K_t$ and $Y_t$ must be larger than the natural rate of growth.

Therefore, our steady state analysis has already shown that the economic growth is gradually reduced from higher level (higher than the natural rate of growth) to the lower level (equal to the natural rate of growth) during the development process (from system 1 to system 2 and 3).

5 The Dynamics of Technology

In this section, we shall analyze the dynamics of $x_t$ and $h_t$ by relying on (42) and (43). As we have pointed earlier, the dynamics of $x_t$ and $h_t$ is somehow autonomous to the whole system (35) - (43).

5.1 The Steady States of $h_t$

The following is the proposition with regard to the steady state of $h_t$.

**Proposition 4** For the autonomous dynamic system (43), the steady state of $h_t$ can be expressed as follows:

\(^{12}\)The definition of natural rate of growth was first given in Harrod (1939).
1. When $\theta_a < x_f$, there are possibly two steady states denoted as $(\bar{h}_1, \bar{h}_2)$ with $\bar{h}_1 = 0$ and $\bar{h}_2 = \frac{1}{(1 - \epsilon)} + \frac{x_f - \theta_a}{\theta_f(1 - \epsilon)}$ (59)

where $\bar{h}_2$ lies in the range $(\frac{1}{1-\epsilon}, +\infty)$

2. When $\theta_a > x_f$, the system has only one steady state, which is 0.

The proof of this proposition is given in the appendix.

The meaning of this proposition can be expressed by relying on Figure 6. Note that the two functions $f(h)$ and $g(h)$ in Figure 6 is defined to be

$$f(h) = \frac{(1 + x_f)h}{1 + \theta_f(1 - \epsilon)h + (\theta_a - \theta_f)}$$

$$g(h) = \frac{1 + x_f h}{1 + \theta_a h}$$

Therefore, equation (43) can be written as

$$h_t = \begin{cases} 
  f(h_{t-1}), & \text{if } h_{t-1} > \frac{1}{1-\epsilon} \\
  g(h_{t-1}), & \text{otherwise}
\end{cases}$$

Thus, $h_t$, given $h_{t-1}$, can be completely described by two separated functions $f(\cdot)$ and $g(\cdot)$ intersected at $1/(1 - \epsilon)$. The dashed lines in the figure are the extensions of $f(\cdot)$ and $g(\cdot)$, but does not reflect $h_t$. Depending on whether $\theta_a < x_f$, we have drawn two curves of $f(\cdot)$ and two lines of $g(\cdot)$ from up to the bottom, corresponding to the two situations $\theta_a < x_f$ and $\theta_a > x_f$. Thus,
when \( \theta_a < x_f \), there are two steady states \((\tilde{h}_1, \tilde{h}_2)\) as in the proposition; when \( \theta_a > x_f \), there is only one steady state, which is 0 as in the proposition. Also note that since \( \epsilon \in (0, 1) \), \( \tilde{h}_2 > 1 \) when \( \theta_a < x_f \).

5.2 The Dynamics of \( h_t \)

Next, we discuss the dynamics of \( h_t \). Figure 7 and 8 provide the trajectories of \( h_t \) for the two cases of \( \theta_a < x_f \) and \( \theta_a > x_f \) respectively. Due to the relatively backward technology, it is reasonable to assume that the initial condition of \( h_t \), denoted as \( h_0 \), is higher than \( 1/(1 - \epsilon) \) as in the figures. As can be found there, if \( \theta_a > x_f \), \( h_t \) will eventually move to 0 since it is the only steady state (see Figure 7). On the other hand, if \( \theta_a < x_f \), the trajectory of \( h_t \) will move to \( \tilde{h}_2 \) (see Figure 8).

We remark that by definition \( h_t \) is the proportion of frontier technology over domestic technology, that is, \( A_f^t / A_t \). Thus \( h_t \to 0 \) indicates that the domestic technology \( A_t \) will gradually pass the frontier technology \( A_f^t \) and continue to pull the distance. On the other hand, from proposition 3, \( \tilde{h}_2 \) is significantly larger than 1. Thus, \( h_t \to \tilde{h}_2 \) indicates that the domestic technology \( A_t \) is significantly less than frontier technology.

Note that whether \( h_t \to 0 \) or \( h_t \to \tilde{h}_2 \) purely depends on whether \( \theta_a > x_f \), or whether the growth rate of technology due to the domestic R&D is larger than frontier’s growth rate. We thus find that if \( \theta_a > x_f \), or the growth rate of technology due to the domestic R&D is larger than frontier’s growth rate, the less developed country will eventually catch up the technology in the frontier coutry, or \( h_t \to 0 \). Otherwise, \( h_t \) will stagnate at \( \tilde{h}_2 \), indicating
that the domestic country will never have a chance to catch up the technology in the frontier country.

5.3 The Dynamics of $x_t$

Given the discussion on the dynamics of $h_t$, we are now able to discuss the dynamics of $x_t$ by relying on (42). We have already known that if $\theta_a < x_f$, $h_t \rightarrow \tilde{h}_2$. From (42), this also means that $x_t \rightarrow \theta_f(1-\epsilon)\tilde{h}_2 - \theta_f + \theta_a$. Using (59) in Proposition 4 to explain $\tilde{h}_2$, we find that $x_t \rightarrow x_f$. On the other hand, if $\theta_a > x_f$, we find from (42) that $x_t \rightarrow \theta_a$. This discussion allows us to write the steady state of $x_t$ as follows:

$$\bar{x} = \begin{cases} 
\theta_a, & \text{if } \theta_a > x_f \\
x_f, & \text{otherwise}
\end{cases} \quad (60)$$

The economic meaning of (60) can be expressed as follows. First, in the case of $\theta_a > x_f$, the domestic technology will eventually pass over the technology in frontier country even without importing technology. Indeed, importing technology will become zero as long as the domestic technology reaches to certain proportion of frontier technology, that is, $A_t/A_{f,t} > (1-\epsilon)$. Thus domestic technology will grow at $\theta_a$.

Next, we shall consider the case of $\theta_a < x_f$. In the first place, $\bar{x}$ cannot be higher than $x_f$. Consider if $\bar{x} > x_f$. In this case, the domestic technology $A_t$ will eventually surpass the frontier technology $A_{f,t}$. When $A_t$ surpass $A_{f,t}$, importing technology will be 0, and thus $x_t$ will be reduced to $\theta_a$, which is
less than $x_f$. We thus find that $\bar{x}$ cannot be higher than $x_f$ in the steady state. Secondly, $\bar{x}$ also cannot be lower than $x_f$. Consider if $\bar{x} < x_f$. In this case, the technological distance becomes larger and larger. When the distance become larger, importing technology will be more, and thus $x_t$ will increase. We thus find that $\bar{x}$ cannot be lower than $x_f$. All these indicate that at the steady state, $\bar{x}$ has to be equal to $x_f$. In particular, the equilibrium condition must appear to be the co-existence of two types of technological progress: the progress from domestical R&D, which is $\theta_a$, and the progress from importing technology, which is equal to $x_f - \theta_a$.

Combined with the previous discussion, we find that if $\theta_a < x_f$, that is, if the growth rate of technology from domestic R&D is less than the growth rate of frontier technology, the less-developed country will never be able to catch up with the frontier country in technology even if its technology is growing at the same rate as frontier country.

6 The Development Process

Given the determination of $x_t$, we are now able to analyze the model.

6.1 The Parameters

Since the model has high dimensions, (4 dimensions for system 2 and 3), a formal mathematical analysis on the dynamic property of our model seems intractable. Therefore, we mainly rely on a numerical method to detect the dynamic property of our model. For this, we shall first specify the parameters used in our numerical analysis (see Table 2).

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>0.01</th>
<th>$\beta_a$</th>
<th>0.345417</th>
<th>$p^*$</th>
<th>0.03</th>
<th>$N^b$</th>
<th>0.7</th>
<th>$\theta_a$</th>
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<td>$\alpha_w$</td>
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<td>$B$</td>
<td>0.655967</td>
<td>$N^*$</td>
<td>0.9</td>
<td>$\theta_f$</td>
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<td>$\xi_m$</td>
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<td>$\alpha_p$</td>
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<td>$s$</td>
<td>0.4</td>
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<td>$\epsilon$</td>
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<tr>
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<td>$\alpha_x$</td>
<td>0.433629</td>
<td>$l$</td>
<td>0.01</td>
<td>$\alpha$</td>
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<td></td>
</tr>
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<td>$d$</td>
<td>0.8</td>
<td>$\kappa$</td>
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<td>$x_f$</td>
<td>0.02</td>
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</table>

Most of the parameters used in this paper comes from Gong (2013) where the behavior functions of (29), (30) and (31) are estimated by employing the annual data from China. Slight adjustment is applied here to make the derived steady state more suitable. The other parameters in the table are either estimated with the method of moments or simply specified when the corresponding data is not available. It should be noted that although we provide some estimations (which is rather unsophisticated due to the lacking
of the data source in China), the followed analytical result should not be sensitive to the possible bias of our estimated parameters as long as they are in the economically meaningful regions. In addition, we shall remark that the benchmark case $\theta_a < x_f$ indicates that the growth rate of technology from domestic R&D is less than the growth rate of frontier technology.

Given the parameters as recorded in Table 2, the steady states are recorded in Table 3.

<table>
<thead>
<tr>
<th>system</th>
<th>$i$</th>
<th>$\bar{p}$</th>
<th>$\bar{w}$</th>
<th>$\bar{m}$</th>
<th>$\bar{N}$</th>
<th>$k, \bar{y}$</th>
<th>$h$</th>
<th>$x$</th>
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<td>0.03</td>
<td>0.0094</td>
<td>0.2033</td>
<td>N.A.</td>
<td>0.0813</td>
<td></td>
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</tr>
<tr>
<td>2</td>
<td>0.1102</td>
<td>0.03</td>
<td>N.A.</td>
<td>0.0960</td>
<td>N.A.</td>
<td>0.0302</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
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<td>0.03</td>
<td>0.1453</td>
<td>0.0960</td>
<td>0.976</td>
<td>0.0302</td>
<td>4.8951</td>
<td>0.02</td>
</tr>
</tbody>
</table>

6.2 Simulating the Development Process

As an exercise, we first provides a simulation to the system 1 when $N_t$ is assumed to have no feedback effect on the price and wage, that is, $p_t$ and $w_t$ follow (44) and (45) respectively. In addition, we shall assume that the growth rate of technology is fixed at its steady state so that we are focused only on the issue of switching out from system 1. The initial condition is all at 5% less than the corresponding steady states while $N_0$ is set at 0.40. As we can find in Figure 9, though the system is asymptotically stable in the space of $(p_t, i_t, m_t, w_t, y_t)$, the employment $N_t$ is increasing continuously. Therefore, the economy will eventually be switched out from system 1.

Figure 10 provides the simulation for the development process with $N_t$ going through different ranges. The initial conditions are expressed as follows. First, we set the initial condition $N_0$ at 0.4, indicating that the economy starts at a less-developed dual economy. Second, we set the initial condition $h_0$ at 50, which represents roughly the ratio of technology in U. S. over the technology in China at the begining of 80’s. Given this $h_0$, the initial condition $x_0$ is set about 0.08 by following (43). All the other initial conditions are the same as in Figure 9.

As can be found in the figure, around 40 years, significant changes do occur in the structure of the economy: investment, money and output growth experiences a sharp decline (see Panel A, C and F respectively); wages starts to cyclically increase (see Panel B), the employment increase becomes slowdown (see Panel D) and inflation rate become more fluctuated (see Panel E). All these seems to suggest that the second stage of economic development starts around 40 years.
6.3 Understanding the Mechanism of Development Process

In a market economy, there are two distinctive forces, demand and supply, in determining output. Given supply (or production capacity as defined in our model), output is determined by demand. But if the output determined by demand is too close to the given supply, inflation might arise. This indicates that supply only provides the possibility of economic growth while growth must be realized through the demand.

If we consider in this way, we find that investment play a key role in the development process. As we have discussed previously, investment creates not only the demand via multiplier but also the production capacity via accumulating capital stock (i.e., building new production lines, new plants and new facilities). For plants and facilities to work, more labors are needed to be attracted into cities, while the existence of huge surplus labor in the earlier stage of economic development (i.e., in system 1) means that the labor supply does not pose a constraint. Thus, the high growth of output $y_t$ in the earlier stage of economic development is primarily driven by the high growth of investment $i_t$.

However, with the economic development, more and more surplus labors have been absorbed into the cities and thus the employment is increasing.
Figure 10: The Development Process
When the employment is above $N^b$, which is 0.7 in this paper, it has a feedback effect on wage $w_t$ and thus on inflation $p_t$. The higher inflation rate $p_t$ will cause the monetary authority to shrink the money supply $m_t$, and thus places a negative effect on investment (see the investment function (36)).

Therefore, the development process can be revealed not only as the gradual increase in employment rate $N_t$ (see Panel D in Figure 10), but also as the increase in the growth rate of wage $w_t$ (see Panel B) and decrease in growth rate of money supply $m_t$ (see Panel C), while inflation rate $p_t$ can be kept around the target level $p^*$ due to the monetary policy (see Panel E). All these expresses the process of investment rate $i_t$ and the growth rate of output $y_t$ from higher to lower levels especially after $N^b$ is passed (see Panel A and Panel F respectively).

6.4 Kuznets Curve

One of the important properties of development process that has been discussed frequently in literature is the Kuznets curve. Kuznets (1955) discovered that in the development process of an economy, income distribution will first get worse and then gradually improve. The whole process is likely to be a U-shaped curve. Here we try to show that our model can also generate a similar curve as to Kuznets (1955) in the development process of our model economy.

Assume that we can use the wage share as a measurement of income distribution: income distribution gets worse if wage share of total income is decreased.\(^{13}\) Let $\omega_t$ denote the wage share over total income. Thus, by definition,

$$\omega_t \equiv \frac{W_tL_t}{P_tY_t}$$

where $W_t$ and $P_t$ are the wage and price level in nominal term. Using (25) to express $L_t$ in the above, we obtain

$$\omega_t = \frac{W_t(U_t)^{\frac{1-\alpha}{\alpha}}}{A_tP_t}$$

Note that from (21) - (23), $U_t = \frac{i_t}{sB}$, thus the above equation can further be written as

$$\omega_t = \frac{(1 + w_t)}{(1 + x_t)(1 + p_t)} \left( \frac{i_t}{i_{t-1}} \right)^{\frac{1-\alpha}{\alpha}} \omega_{t-1}$$

\(^{13}\)We shall remark that such a way of measuring income distribution is more classical as in Ricardo and Marx, among others. It is often called the functional income distribution.
Clearly, whether the wage share \( \omega_t \) will increase or decrease depends on how fast the wage increases relative to the increase in price among the others.

Let \( q_t \) denote the per capita output so that \( q_t \equiv \frac{Y_t}{L_t} \). It is not difficult to find that

\[
q_t = \frac{1 + y_t}{1 + \frac{y_t}{q_{t-1}}} \tag{61}
\]

where \( y_t \) is given by (41).

In Figure 11, we illustrate the trajectories of wage share \( \omega_t \), per capita output \( q_t \) and the Kurznets curve, the wage share \( \omega_t \) in response to per capita output \( q_t \) in the development process. The initial conditions of this exercise are the same as in Figure 10. As can be found there, we do find a U-shaped curve as proposed by Kurznets (1955).

### 6.5 The Middle Income Trap

To examine the middle income trap, we assume that the frontier economy grows at its steady state so that its per capita GDP is growing at \( x_f \), the growth rate of technology:

\[
q^f_t = (1 + x_f)q^f_{t-1} \tag{62}
\]
On the other hand, the per capita GDP in domestic country $q_t$ is governed by (61). Define $H_t \equiv \frac{q_t}{q_t^f}$ so that $H_t$ is the ratio of per capita GDP in frontier economy to developing economy. We thus can obtain the mathematical definition of middle income trap as follows:

**Definition 5** The so-called middle income trap is referred to the situation in which a less developed country cannot continue to grow faster after experiencing a period of faster growth. This makes its distance in GDP per capita from frontier economy no longer shrink. Mathematically, this can be written as

$$\lim_{t \to +\infty} H_t = \bar{H}$$

where $\bar{H}$ is significantly larger than 1.

We remark that this definition with regard to middle income trap is often named as relative definition. Now using (61) and (62) to express $q_t$ and $q_t^f$ in the definition, we find that

$$H_t = \frac{(1 + x_f)(1 + l)}{1 + y_t} H_{t-1}$$

From (58), we find that if the economy is successfully switched out from system 1 (or the first stage of economic development), the economy will eventually grow at the natural rate so that in the long run, the dynamics of $H_t$ follows

$$H_t = \frac{1 + x_f}{1 + \bar{x}} H_{t-1}$$

Applying (60) to the above equation, we obtain

$$\bar{H} = \begin{cases} 0, & \text{if } \theta_a > x_f \\ H^*, & \text{otherwise} \end{cases}$$

This indicates that in our benchmark case, $\theta_a < x_f$, the trajectory $H_t$ will rest at a certain point $H^*$ that could be larger than 1. The value of $H^*$ depends on the initial condition $H_0$. Since $H_0$ represents the ratio of per capita GDP of frontier economy over domestic economy, it is almost certain that $H_0$ will be higher enough to make the trajectory of $H_t$ rest at the $H^*$ that is significantly larger than 1. Figure 12 illustrates the dynamics of $H_t$ among others. The initial condition of this figure is the same as in Figure 10.

---

14Indeed, as point out by Han and Wei (2015), absolute middle income trap does not exist as long as the growth rate of per capita GDP is positive in the long run.

15It is possible that the rest point could be less than 1 if the initial $H_0$ is less than 1 in our dynamic system.
As can be found in Panel A of the figure, the growth rate of technology $x_t$ starts at the initial level of 8.2%, and then gradually decline to its steady state 2% which is $x_f$, the growth rate of frontier technology. Yet, the growth rate from importing technology is gradually from original 6.7% to eventually 0.05 (see Panel B). Therefore, the steady state $\bar{x}$ equal to 2 is made up by $\theta_a$ (which is 1.5%), plus 0.05% from importing technology. We find that such a growth pattern in technology will make $h_t$, the ratio of technology stop declining at the steady state $\bar{h}_2$, which is 4.8951 (see Table 3). Given such distance in technology, the ratio of per capita GDP $H_t$ stops declining at 3.7. Therefore, the economy gets into the middle income trap.

Note that this benchmark case assume that $\theta_a = 0.015$ so that it is less than $x_f$. Now let us consider $\theta_a = 0.025$. Apparently, in this case, $\theta_a > x_f$. Figure ?? illustrates the dynamics of $H_t$ among others. The initial condition is the same as in Figure 12.

As can be found there, the growth rate of technology $x_t$ stop declining at its steady state 2.5%, which larger than $x_f$ (see Panel A) while importing technology is eventually 0. (see Panel B). This make the ratio of technology $h_t$ and the ratio of per capita GDP $H_t$ both tend to 0. Therefore, the economy surpass the frontier economy and get over the middle income trap.

Figure 12: Middle in Trap, the Benchmark Case
Figure 13: Getting over the Middle Income Trap
7 The Discussion: New Normal in Chinese Economy

This paper suggests that the development process of a less-developed country can be divided into two stages with significant different properties in terms of structural endowments, production model, income distribution, development level, driving force of economic growth, among many others. The two stages of economic development have been indicated in the growth theory of macroeconomics and in the variety of "turning point" theories in development economics, including Lewis’s dual economy, Kuznets curve and middle income trap. We also construct the model of development process that reveals these two stages.

The Chinese economy has assumed entered into a stage of "new normal". An interesting question is then whether the so-called "new normal" can be understood as the economy being finished (or almost finished) its first stage of economic development?

In the first place, GDP per capita in China has reached to 6700 US dollars in 2013. According to international standard, China has already been a middle income country.

Second, after 30 years of high growth, the large scale of the surplus labor in China no longer exists. In reality, "recruitment difficult" and "recruitment director" has appeared from time to time in the eastern coastal area. A related study shows that China’s surplus labor force has been reduced from 98 millions in 1990 to 42.67 millions in 2012.\(^6\) Although this number is still so huge to many countries, it makes China’s current overall unemployment rate about 6.6%. This unemployment rate has already been less than those in European countries (such as Germany and France)

Third, the shortage of surplus labor also means that wages are rising faster. Since 2011, the wage in the urban private sector has increased at the speed higher than the GDP growth. The rapid wage growth has also led to a reversal of the proportion of wage income over GDP. This also means that China’s income distribution began to improve, that is, the turning point in Kuznets curve has emerged.

Fourth, as a developing country, China’s economy has been largely characterized by capital intensive. For example, China’s steel production has been the world’s first for more than 10 years, while the excessive capacity is also concentrated in the capital intensive industries (such as steel industry, etc.).

To sum up, the Chinese economy has entered the second stage of its

\(^6\)See Xu (2015).
economic development. Therefore, if we use "new normal" to describe the current Chinese economy, it must refer to the second stage of economic development in China. However, getting into the second stage or finishing with the first stage does not necessarily mean that China will automatically get out of middle income trap. As we have demonstrated in this paper, this will depend on how fast the technological progress that China can make. Since it becomes more and more difficult for China to import technology, the source of technological progress can only rely on its own research and development. For China to eventually catch up a developed country at the forefront in terms of GDP per capita, the growth rate of technology through its own R&D must be higher than the same rate in the related developed country at the forefront. This is the most important challenge for China in the future.

8 Appendix

8.1 Proof of Proposition 1

Expressing $I_{j,t+k}$ in (9) in terms of (10) and $c_{j,t+k}$ in terms of (7), we find that the problem (9) can be re-written as

$$\max E \sum_{k=0}^{\infty} \beta^k \{P_{j,t+k}Y_{j,t+k} - P_{t+k}c_{j,t+k}(U_{j,t+k})Y_{j,t+k} - (1 + r)P_{t+k}[K_{j,t+k} - (1 - d_j)K_{j,t+k-1}]\}$$

Note that from (3) we find that $U_{j,t+k}$ is also a function of $K_{j,t+k-1}$. In particular,

$$\frac{\partial U_{j,t+k}}{\partial K_{j,t+k-1}} = -\frac{Y_{j,t+k}}{B(\mathcal{K}_{j,t+k-1})^2} = -\frac{B(U_{j,t+k})^2}{Y_{j,t+k}}$$

Therefore, the problem becomes the choice of the sequence $\{K_{j,t+k}\}_{k=0}^{\infty}$. The Euler equation for this problem can be written as

$$E\beta P_{t+k} \left[\frac{(1 - \alpha)\omega}{\alpha} (U_{j,t+k})^{1-\alpha-1} B (U_{j,t+k})^2 - v_j (U_{j,t+k})^-2 (U_{j,t+k})^-2 + (1 + r)(1 - d_j)\right]$$

$$- P_{t+k-1}(1 + r)\beta^{k-1} = 0$$

which can further be simplified as

$$E\beta \pi \left[\frac{(1 - \alpha)\omega}{\alpha} (U_{j,t+k})^{\frac{1}{\alpha}} B - v_j (1 + r)(1 + d_j)\right] = 1 + r$$

This equation allows us to obtain

$$(U_{j,t+k})^{\frac{1}{\alpha}} = \frac{(1 + r)/(\beta\pi) - (1 + r)(1 - d_j) + v_j}{\frac{1-\alpha}{\alpha} B\omega}$$
We therefore prove the proposition.

### 8.2 The Proof of Proposition 2

From (21) and (22), we first obtain

\[ Y_t = \frac{1}{s} I_t \quad \text{(63)} \]

Substituting (63) into (23) while recognizing that \( i_t \equiv \frac{I_t}{K_{t-1}} \), we obtain

\[ U_t = \frac{1}{sB} i_t \quad \text{(64)} \]

The investment function (29) can thus be rewritten as (36) as in the proposition. Next, substituting (30) and (31) into (31) and (30) while using (64) to express \( U_t \), we obtain

\[

t = \beta_p + \beta_w (\alpha_w + \alpha_p I_t + \alpha_x i_t) + \frac{\beta_u}{sB} i_{t-1} \\

w_t = \alpha_w + \alpha_p \left[ \beta_p + \beta_w w_t + \frac{\beta_u}{sB} i_{t-1} \right] + \alpha_{n,l} N_{t-1} + \alpha_x x
\]

Re-organizing the above equations, we obtain (37) and (39) as in the proposition.

Next, we shall derive (43) and (42) as in the proposition. By definition, \( h_t \equiv A^f_t / A_t \). Thus when \( A_{f,t-1}(1 - \epsilon) - A_t \), \( h_t > 1/(1 - \epsilon) \). Therefore, the two conditions in (33) and (43) - (42) are equivalent. Using (34) and the first-half of (33) to express \( h_t \), we obtain

\[

h_t = \frac{(1 + x_f) A^f_{t-1}}{\theta_f [A^f_{t-1}(1 - \epsilon) - A_t] + (1 + \theta) A_{t-1}} \\
= \frac{(1 + x_f) h_{t-1}}{\theta_f [h_{t-1}(1 - \epsilon) - 1] + (1 + \theta)}
\]

This the first-half of equation (43) in the proposition. The second half of equation (43) directly comes from the second half of (33) and (34). Similarly, the first half of (33) allows us to obtain

\[
x_t = \theta_f [h_t(1 - \epsilon) - 1] + \theta A_{t-1}
\]

This is the first half of (42) in the proposition, the second half of (42) in the proposition directly comes from the second half of (33).
Expressing $L_t$ in (27) in term of (25), we obtain

$$N_t = \frac{Y_t}{A_t L_t^s} \left( U_t \right)^{\frac{1-\alpha}{\alpha}}$$

Dividing both side by $N_{t-1}$ while expressing $N_{t-1}$ in the right side in term of \( \frac{Y_{t-1}}{A_{t-1} L_{t-1}^s} \left( U_{t-1} \right)^{\frac{1-\alpha}{\alpha}} \), we find that

$$\frac{N_t}{N_{t-1}} = \frac{(1 + y_t) Y_{t-1} A_{t-1} L_{t-1}^s}{(1 + x)(1 + l) A_{t-1} L_{t-1}^s Y_{t-1}} \left( \frac{U_t}{U_{t-1}} \right)^{\frac{1-\alpha}{\alpha}}$$

Substituting (64) into the above, we obtain

$$\frac{N_t}{N_{t-1}} = \frac{(1 + y_t)}{(1 + x)(1 + l)} \left( \frac{i_t}{i_{t-1}} \right)^{\frac{1-\alpha}{\alpha}} \quad (65)$$

Next, we should derive $y_t$. For this, we first divide both sides of (63) by $Y_{t-1}$. This allows us to have

$$1 + y_t = \frac{I_t}{s Y_{t-1}}$$

where $y_t \equiv \frac{Y_t - Y_{t-1}}{Y_{t-1}}$ is the growth rate of output $Y_t$. Note that in the above equation, we can express $I_t$ in terms of $i_t K_{t-1}$. Therefore,

$$1 + y_t = \frac{i_t K_{t-1}/K_{t-2}}{B s Y_{t-1}/(B K_{t-2})} = \frac{i_t (1 + k_{t-1})}{B s U_{t-1}}$$

where $k_t \equiv \frac{K_t - K_{t-1}}{K_{t-1}}$ is the growth rate of capital stock. From (24), one finds that

$$k_t = -d + i_t$$

This is indeed the equation (40) as in the proposition. Now expressing $k_{t-1}$ in terms of $-d + i_{t-1}$ while $U_{t-1}$ in terms of (64), we obtain

$$1 + y_t = \frac{i_t (1 - d + i_{t-1})}{i_{t-1}}$$

(66)
This is indeed the equation (41) as in the proposition. Now substituting (66) into (65) and re-organizing, we obtain (38) as in the proposition. Finally equation (35) in the proposition is simply the same as (28). We thus prove the proposition.

8.3 The Proof of Proposition 3

Let us first consider the range \( h_t > \frac{1}{1-\epsilon} \) so that we can use the first half of (??) to derive the steady state. Assume \( h_t = h_{t-1} = \bar{h} \). Thus, from (??), we obtain

\[
1 = \frac{1 + x^f}{1 + \theta_f(1-\epsilon)\bar{h} + (\theta_a - \theta_f)}
\]

Solving the above equation for \( \bar{h} \), we obtain

\[
\bar{h} = \frac{1}{1-\epsilon} + \frac{x^f - \theta_a}{\theta_f(1-\epsilon)}
\]

This is indeed \( \bar{h}_2 \), which is lying in the range \( \left( \frac{1}{1-\epsilon}, +\infty \right) \) if \( \theta_a < x^f \).

Next, we shall consider the range \( h_t \leq \frac{1}{1-\epsilon} \). In this range, the dynamics of \( h_t \) is governed by the second half of (??). It is not difficult to find that the only steady state is 0 if \( \theta_a \neq x^f \). Otherwise, any number in the range \( [0, \frac{1}{1-\epsilon}] \) can be a steady state at which \( h_t = h_{t-1} \) is satisfied.

References


